

Numerical Simulation of Dynamic Fluid-Solid Coupling of Saturated Soil

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Abstract: This paper confirms that in the u-p coupling model, the calculation results are consistent with the results of the analytical solution. This paper confirms the rationality and feasibility of the u-p-U model solving technology in the u-p-U coupling model. This paper confirms that the calculation results of the two forms are almost the same when $k=10^{-4}$ m/s and $k=10^{-5}$ m/s. When $k=10^{-3}$ m/s the results of the two calculation forms are slightly different. The finite incremental calculus stabilization method can effectively eliminate the phenomenon of numerical oscillation in the region where the pore pressure gradient changes drastically. The finite incremental calculus stability method has a good stability effect in dealing with the dynamic equation of saturated soil which is difficult to calculate.

1. Introduction

At present, the commonly used methods for solving Biot's saturated dynamic equations include semi-analytical method, finite element method, finite difference method and boundary element method [1, 2]. Among them, the semi-analytical method has high computational efficiency in analyzing the dynamic response of semi-infinite saturated soil because it mostly adopts the theoretical derivation methods such as integral transformation, series expansion and potential function decomposition, and does not consider the boundary conditions in the analysis domain. However, it is difficult to apply to irregular structural shapes, complex contact conditions, material nonlinearity and inhomogeneity. The finite element method is widely used in numerical calculation because it can adapt to any complex geometry, material properties and boundary nonlinearity.

Zienkiewicz [3] studied the Biot's saturated dynamic equation by finite element method. According to the basic unknowns and different dynamic processes, the linear and nonlinear

equations of fully coupled u-U, u-p-U and partially coupled u-p are proposed. The porous media theory has been successfully used to analyze the behavior of fluid saturated incompressible porous media. Zhou [4] established a numerical model that can describe the fluid-solid coupling characteristics of saturated deformed low-permeability reservoirs. The coupling numerical method combining finite difference and finite element is used to solve the problem, which proves that the fluid-solid coupling effect of low-permeability reservoirs cannot be ignored. Wang [5] used finite element software to verify the effectiveness of the pore pressure element of the fluid-solid coupling two-phase medium dynamic model in the study of seismic response of saturated soil-underground structure system. Xiuli [6] used finite element software to establish a complete fluid-solid coupling model to simulate the seismic response of the structure buried in saturated soil. He [7] used discrete element software to consider the particle-fluid coupling algorithm of fluid dynamic mesh, and verified the effectiveness of the method. Based on the boundary integral equation, He [8] used the integral transformation method to simulate the wave scattering around the cavity with circular section in saturated soil. Through numerical calculation, Zhang [9] confirmed that the dynamic stress concentration and pore pressure concentration of saturated soil are affected by the degree of fluid-solid coupling, pore compressibility and permeability. Combining the classical elasticity theory, Biot's pore elasticity theory and an effective medium method, Xu [10] proved the importance of considering fluid-solid interaction in seismic metasurface dynamics. Considering the fluid-solid coupling of saturated soil, the stratification of site soil and the three-dimensional propagation of wave, Wang [11] constructed a semi-analytical site dynamic Green's function. Bui [12] proposed a computational framework based on meshless smoothed particle hydrodynamics method, which can be used to study the fluid-solid coupling behavior in deformable porous media. Xiao [13] established an analysis model for fluid-solid coupling problems with discrete solid interactions, and proved the stability and accuracy of the model. Yang [14] established a fluid-solid coupling model for the hydraulic fracturing process of fractured shale, which is of great significance to explain the mechanism of shale hydraulic fracturing. Yao [15] established a fluid-solid coupling heat transfer physical model and proved that the digital simulation method can be used in engineering practice. Tang [16] established a fluid-solid-thermal coupling model of the airship envelope, which can accurately reflect and predict the stress-strain distribution and deformation law of the near-space airship. Based on the multi-field coupling theory, Wang [17] established a fluid-solid-thermal coupling mathematical model of liquid-filled pipes under the combined action of temperature and pressure loads.

This paper mainly focuses on the dynamic fluid-solid coupling response of saturated soil under moving load, and studies the applicability of different dynamic coupling forms of saturated soil and the stability method of dynamic calculation. On this basis, the suitable coupling form is selected to carry out its moving element method.

2. Comparative Analysis of Dynamic Coupling Forms

The interaction between pore water and soil skeleton in saturated soil under dynamic load includes inertia, viscosity and mechanical coupling. The permeability coefficient and load frequency are the main factors affecting the coupling effect between soil skeleton and pore water. Because the relative acceleration of fluid to solid phase is neglected in the simplified form of u-p, it is necessary to measure the difference between the two dynamic coupling forms under different load frequencies and different permeability coefficients. The model has a horizontal calculation length of 20m and a height of 3m. The load moves in the horizontal direction at a speed of V . The load amplitude is 20kPa and the distribution width is 0.4m. The upper surface is a free permeable boundary. The soil calculation parameters are $E=250$ MPa, $\nu=0.2$, $\rho_s=2000\text{kg/m}^3$, $k=10\text{-}12\text{m}^2$

10-11 m², 10-10 m², $Q=2 \times 10^9$ Pa, $n=0.3$, $\alpha=1$. Fig 1 is the comparison of the calculation results of two forms of pore pressure under different permeability coefficients. It can be seen from the figure that the calculation results of the two forms are almost the same when $k=10^{-4}$ m/s and $k=10^{-5}$ m/s. The main reason is that when the permeability coefficient is small, the relative displacement between pore water and solid skeleton is relatively small, and the inertial coupling effect is relatively weak. When $k=10^{-3}$ m/s, the results of the two calculation forms are slightly different, and the calculation results of u-p-U coupling model are larger. When the permeability coefficient is large, the relative flow resistance between the pore water and the solid skeleton is small, and a large relative displacement is formed between the two, thus showing obvious inertial coupling effect. It can be considered that the inertial coupling effect can be ignored when the permeability coefficient $k < 10^{-3}$ m/s. Fig 2 shows the difference between the pore pressure time history curves of the two coupling forms at different speeds. The calculation rules of the two are consistent. However, as the speed increases, the peak difference between the two coupling forms increases, but the difference is not obvious. It can be seen that the error caused by the dynamic fluid-solid coupling effect of coarse-grained soil under moving load is small, and the u-p model can significantly reduce the degree of freedom and improve the calculation efficiency. Therefore, the u-p form can be used to calculate the dynamic response of saturated subgrade surface.

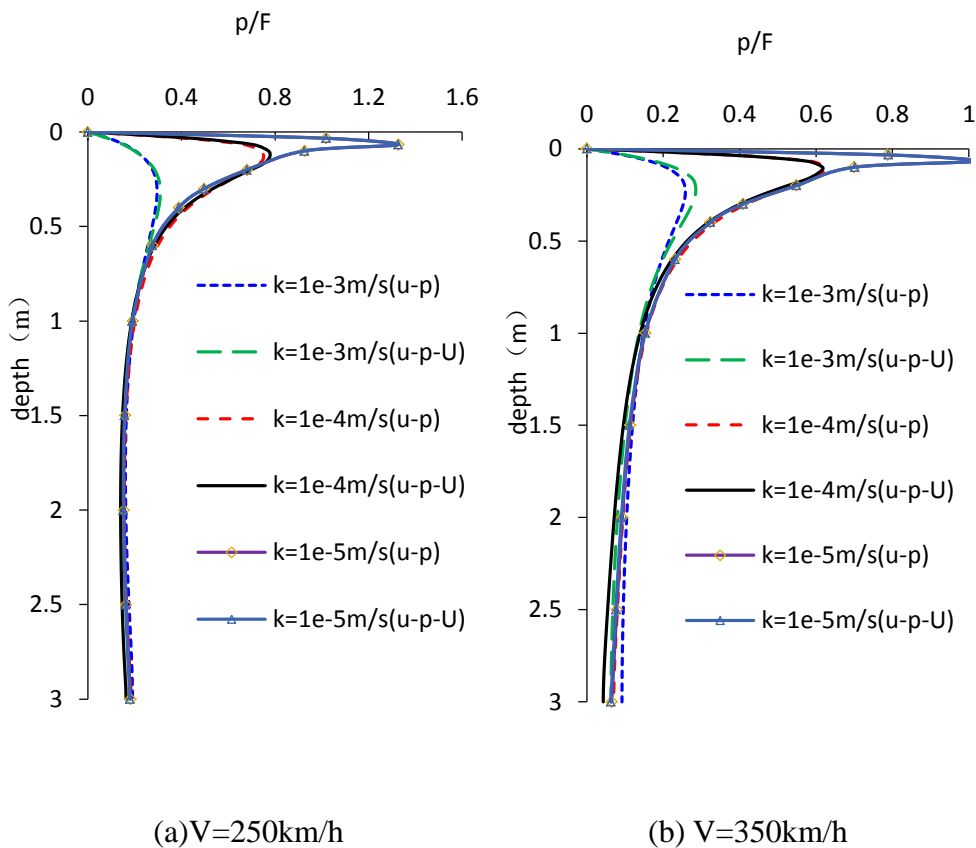


Figure 1. The calculation results of two forms of pore pressure under different permeability coefficients

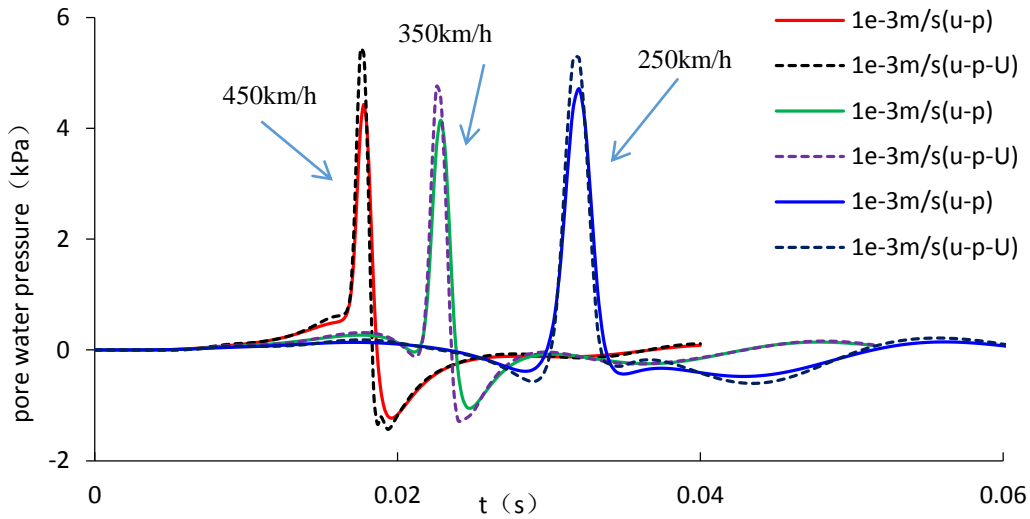


Figure 2. The calculation results of two forms of pore pressure at different velocities

3. Analysis of Stabilization Methods

When the finite element method is used to calculate multiple field coupling equations, the Babuska-Brezzi compatibility condition should be satisfied to ensure the computational stability[18]. For Biot’s dynamic equation, the order of solid displacement variable element is at least one order higher than that of fluid pressure element. When the pore water is incompressible and the permeability coefficient is low, the equivalent stiffness matrix formed by Biot’s dynamic equation is ill-conditioned and prone to non-convergence. In addition, the traditional Galerkin finite element method has good applicability in dealing with the standard wave equation, but in dealing with the diffusion equation, the numerical vibration is easy to occur in the place where the gradient is relatively large, which leads to the calculation distortion. Therefore, the necessity of Babuska-Brezzi compatibility condition and the reliability of finite incremental calculus method to improve the calculation stability are illustrated by numerical examples.

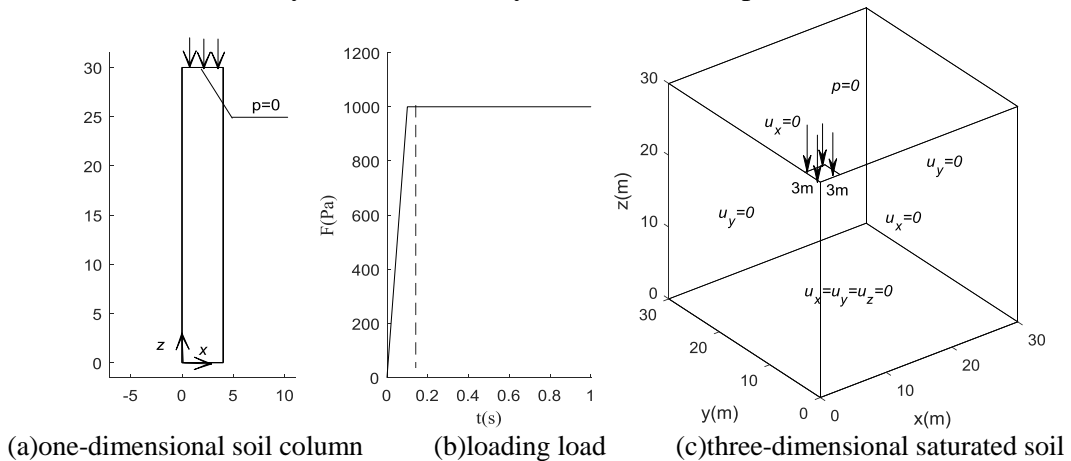


Figure 3. Computational model

One-dimensional saturated soil column and three-dimensional saturated soil under step load and harmonic load are selected for analysis. As shown in Fig 3, the step load increases to a stable amplitude within the first 0.1s (Fig 3 (b)). In order to quantitatively analyze the load amplitude $F=1$ kPa, the harmonic load expression is $F=1+0.5\sin(2t)$ kPa, focusing on the influence of permeability

coefficient and pore water compression characteristics on different calculation methods. The calculation parameters of soil are $E=250$ MPa, $\nu=0.2$, $\rho_s=2000$ kg/m³, $k=10^{-14}$ m²、 10^{-11} m², $Q=9.95 \times 10^{14}$ Pa、 9.95×10^9 Pa, $K_f=2.0$ GPa, $n=0.3$, $\alpha=1$. Three calculation methods are used to compare under different calculation conditions. The first form is that the order of the solid phase displacement variable element is the same as that of the fluid pressure element, which does not meet the Babuska-Brezzi compatibility condition. The corresponding elements of two-dimensional and three-dimensional are u4p4 and u8p8. The second form is that the element order of the solid phase displacement variable is one order higher than that of the fluid pressure element, which satisfies the Babuska-Brezzi compatibility condition. The two-dimensional and three-dimensional corresponding elements are u8p4 and u20p8. The third form is to satisfy the Babuska-Brezzi compatibility condition, and the finite incremental calculus stabilization method is adopted.

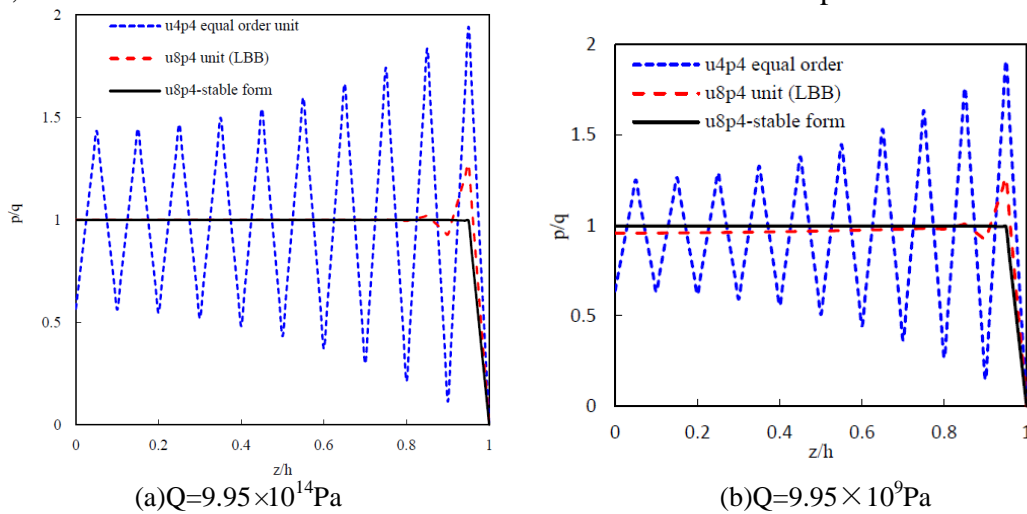


Figure 4. The distribution of pore pressure along the depth of soil column under different unit forms ($t = 1$ s)

Figure 4 is the calculation result of pore pressure of one-dimensional saturated soil under step load. From the diagram, it can be seen that the pore pressure distribution has serious numerical oscillation when u4p4 element is used. The numerical oscillation is the most serious at the upper drainage boundary and propagates along the depth direction. The calculation result of this element is seriously distorted. When u8p4 element is used, the numerical oscillation of pore pressure is obviously suppressed, only a certain numerical oscillation occurs near the drainage boundary, and there is no propagation along the depth direction. When the finite incremental calculus method is used to stabilize the method, the pore pressure distribution along the depth direction is smooth, and there is no numerical oscillation at the drainage boundary. Therefore, the finite incremental calculus stability method can effectively eliminate the phenomenon of numerical oscillation in the area where the pore pressure gradient changes dramatically. In addition, by comparing the different compression properties of pore water, it can be found that when the pore water is considered to be incompressible (Fig 4 (a)), the numerical oscillation of pore pressure is more serious.

Since the u4p4 equal-order element does not meet the LBB condition requirements of the mixed element form, the calculation results are seriously distorted, so the element form should be avoided in the calculation. In order to analyze the advantages of the finite incremental calculus stability method, Fig 5 shows the pore pressure time history curves under different element mesh densities under sinusoidal load. When the permeability coefficient in Fig 5 (a) is small, the upper part can be

considered as impermeable during the analysis period. When the mesh density is coarsened by u8p4 element, the pore pressure time history change is consistent with the load change form and the amplitude remains stable, but the pore pressure amplitude is larger than the applied load amplitude, which reflects that the calculation accuracy is low under this mesh density. When the element mesh density is refined, the calculation accuracy of pore pressure amplitude is continuously improved. When the finite incremental calculus stabilization method is used, a more accurate pore pressure calculation result can be obtained under the coarse mesh density. When the permeability coefficient is large, as shown in Fig 5 (b), the sensitivity of the two calculation results to the grid density is reduced, mainly because the pore pressure gradient does not change drastically under the permeability coefficient. Similarly, when the finite incremental calculus stabilization method is used, higher calculation accuracy can be obtained when the grid density is low. It can be seen that the finite incremental calculus stability method has a good stability effect in dealing with the dynamic equation of saturated soil with difficult calculation (pore water is not easy to compress or permeability coefficient is small).

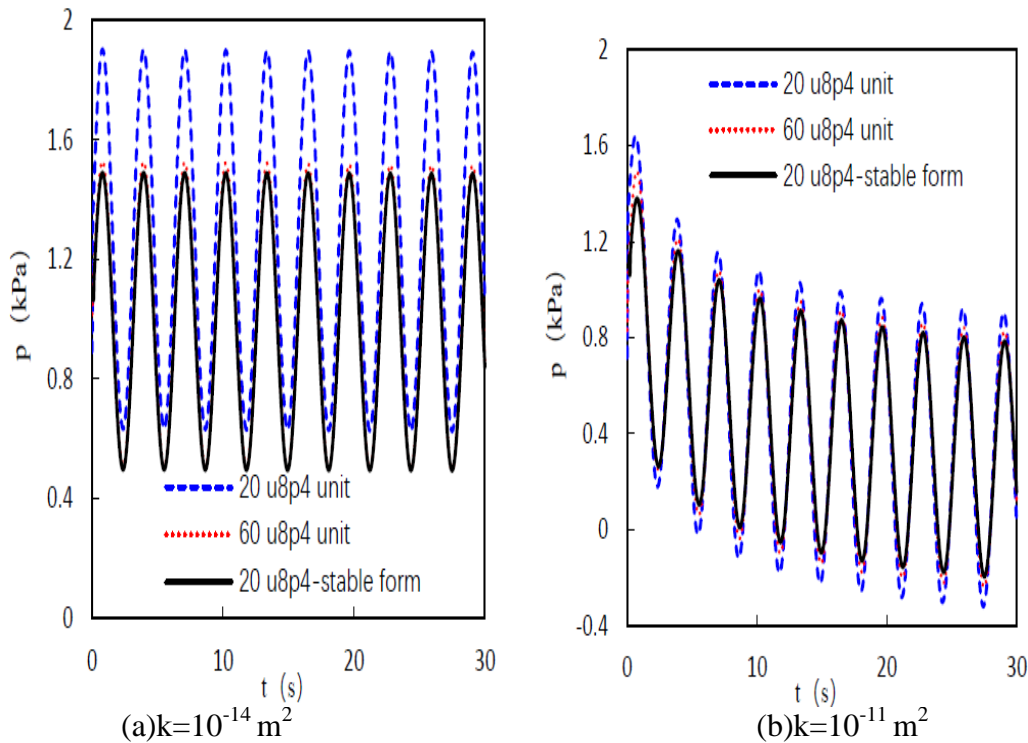


Figure 5. Pore pressure time history curves ($z = 28.5m$) under different mesh densities

Fig 6 shows the spatial distribution of dynamic pore pressure of three-dimensional saturated soil under step load ($t = 0.1s$) when the number of conventional elements is $30m \times 30m \times 30m$. From the diagram, it can be seen that when the finite incremental calculus stability method is used, the dynamic pore pressure distribution is relatively smooth in space, and there is no oscillation where the pore pressure gradient changes greatly. Compared with the stable form, the overall distribution range and law are basically the same when u20p8 element is used. The smoothness of the spatial distribution of dynamic pore pressure is reduced, and the pore pressure amplitude is larger, and there is a slight oscillation in the place where the pore pressure gradient changes greatly. When u8p8 element is used, the dynamic pore pressure produces self-locking phenomenon, and the pore pressure amplitude is much higher than the previous two forms, which causes serious numerical oscillation in the depth direction, resulting in distortion of the calculation results.

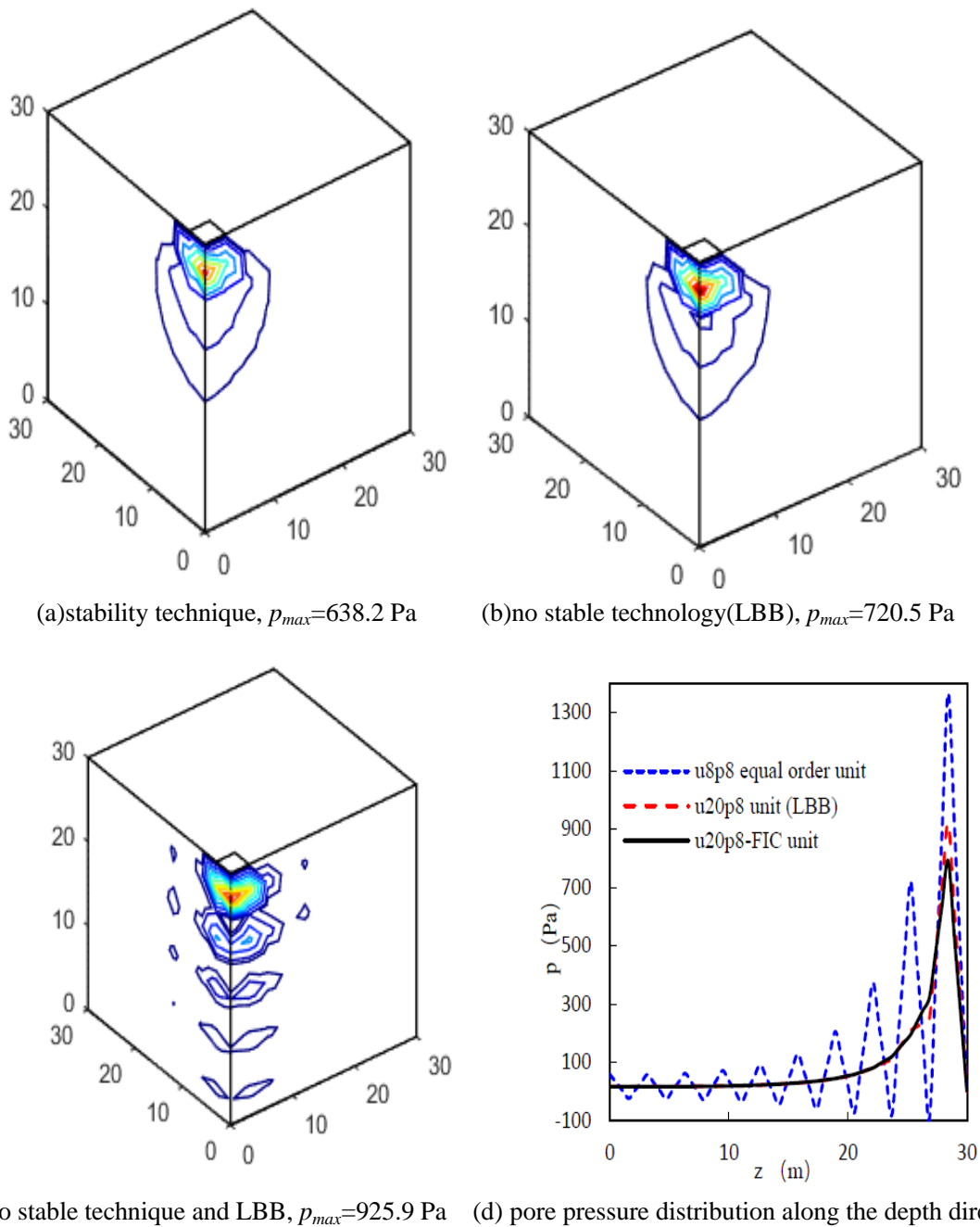
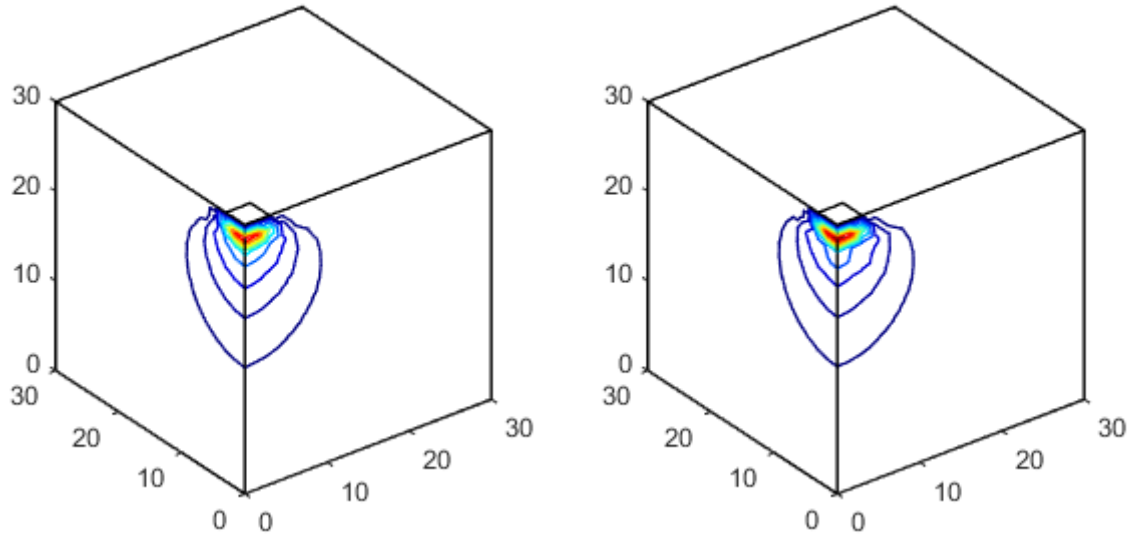


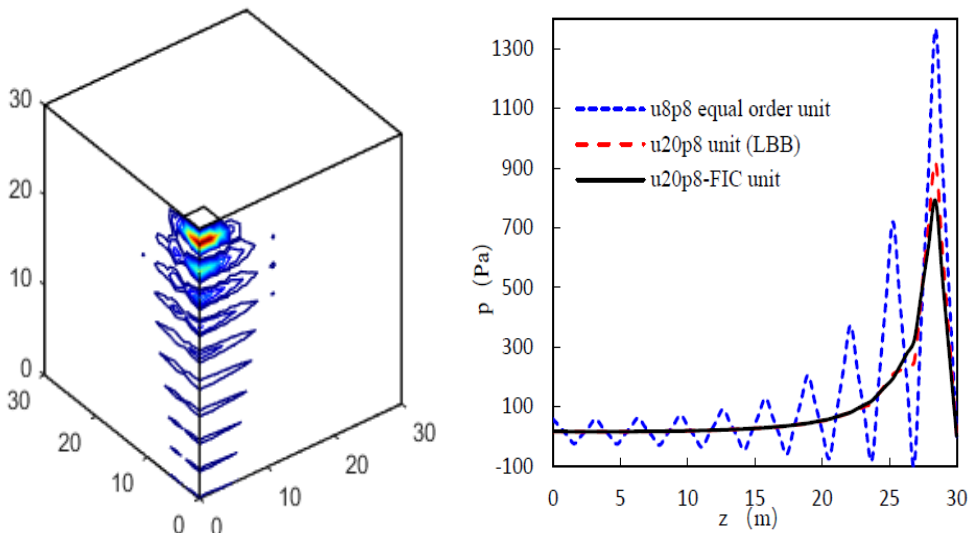
Figure 6. The distribution of pore pressure under different unit forms (2640 units, $t = 0.1s$)

Fig 7 shows the spatial distribution of dynamic pore pressure of three-dimensional saturated soil under step load ($t = 0.1s$) when the element is refined, and the number of elements is about 6 times of the above number. It can be seen from the figure that when the finite incremental calculus stabilization method and u20p8 element are used, the pore pressure distribution in space is relatively uniform and continuous, and the smoothness is obviously improved. It can be seen that the finite incremental calculus stabilization method also has a certain dependence on the element size. In addition, the pore pressure amplitude calculated by u20p8 element is larger, and there is a slight oscillation in the place where the pore pressure gradient changes greatly. When u8p8 element is used, the self-locking zone of pore pressure is more concentrated. The calculated pore pressure amplitude is higher, the numerical oscillation phenomenon is more serious, and the calculation

results are seriously distorted.



(a) stabilization technology, $p_{max}=790.7$ Pa (b) no stable technology (LBB), $p_{max}=920.8$ Pa



(c) no stable technique and LBB, $p_{max}=1365.2$ Pa (d) Pore pressure distribution along the depth direction

Figure 7. The distribution of pore pressure under different unit forms (15770 units, $t = 0.1s$)

4. Conclusions

In this paper, the correctness of the u-p and u-p-U dynamic coupling models is verified, and the analytical solutions of the dynamic response of one-dimensional incompressible saturated soil given in the literature are compared. This paper measures the difference between the two dynamic coupling forms under different load frequencies and different permeability coefficients. In this paper, numerical examples are given to illustrate the necessity of Babuska-Brezzi compatibility condition in the solution of Biot's dynamic equation and the reliability of finite incremental calculus method in improving the computational stability.

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Data Availability

Data sharing is not applicable to this article as no new data were created or analysed in this study.

Conflict of Interest

The author states that this article has no conflict of interest.

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